Hybrid protoneutron stars within a static approach.

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We study the hadron-quark phase transition in the interior of protoneutron stars. For the hadronic sector, we use a microscopic equation of state involving nucleons and hyperons derived within the finite-temperature Brueckner-Bethe-Goldstone many-body theory, with realistic two-body and three-body forces. For the description of quark matter, we employ the MIT bag model both with a constant and a density-dependent bag parameter. We calculate the structure of protostars within a static approach. In particular we focus on a suitable temperature profile, suggested by dynamical calculations, which plays a fundamental role in determining the value of the minimum gravitational mass. The maximum mass instead depends only upon the equation of state employed.

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I. INTRODUCTION

After a protoneutron star (PNS) is successfully formed in a supernova explosion, neutrinos are temporarily trapped within the star (Prakash et al. 1997). The subsequent evolution of the PNS is strongly dependent on the stellar composition, which is mainly determined by the number of trapped neutrinos, and by thermal effects with values of temperatures up to 30-40 MeV (Burrows & Lattimer 1986; Pons et al. 1999). Hence, the equation of state (EOS) of dense matter at finite temperature is crucial for studying the macrophysical evolution of protoneutron stars.

The dynamical transformation of a PNS into a NS could be strongly influenced by a phase transition to quark matter in the central region of the star. Calculations of PNS structure, based on a microscopic nucleonic equation of state (EOS), indicate that for the heaviest PNS, close to the maximum mass (about two solar masses), the central particle density reaches values larger than $1/\mathrm{fm}^3$. In this density range the nucleon cores (dimension ≈ 0.5 fm) start to touch each other, and it is likely that quark degrees of freedom will play a role.

In this work we will focus on a possible hadron-quark phase transition. In fact, as in the case of cold NS, the addition of hyperons demands for the inclusion of quark degrees of freedom in order to obtain a maximum mass larger than the observational lower limit. For this purpose we use the Brueckner-Bethe-Goldstone (BBG) theory of nuclear matter, extended to finite temperature, for describing the hadronic phase and the MIT bag model at finite temperature for the quark matter (QM) phase. We employ both a constant and a density-dependent bag parameter B. We find that the presence of QM increases the value of the maximum mass of a PNS, and stabilizes it at about 1.5–1.6 $M_\odot$, no matter the value of the temperature.

The paper is organized as follows. In section II we present a new static model for PNS. Section III is devoted to the description of the hadron-quark phase transition within the EOS mentioned above. In section IV we present the results about the structure of hybrid PNS and, finally, we draw our conclusions.

II. A STATIC MODEL FOR PNS

Calculations of static models of protoneutron stars should be considered as a first step to describe these objects. In principle the temperature profile has to be determined via dynamical calculations taking into account neutrino transport properly [1]. Many static approaches have been developed in the past decade [2][3], implementing several finite temperature EOS and assuming an isentropic or an isothermal [4] profile throughout the star.

In our model we assume that a PNS in its early stage is composed of a hot, neutrino opaque, and isothermal core separated from an outer cold crust by an isentropic, neutrino-free intermediate layer, which will be called the envelope throughout the paper.

A. Isothermal core

For a PNS core in which the strongly interacting particles are only baryons, its composition is determined by requirements of charge neutrality and equilibrium under weak semileptonic processes, $B_1 \to B_2 + l + \bar{\nu}_l$ and $B_2 + l \to B_1 + \nu_l$, where B_1 and B_2 are baryons and l is a lepton (either an electron or a muon). Under the condition that neutrinos are trapped in the system, the beta equilibrium equations read explicitly

$$\mu_i = b_i \mu_n - q_i(\mu_l - \mu_{\nu_l}) , \qquad (1)$$

where b_i is the baryon number, and q_i the electric charge of the species i. Because of trapping, the numbers of leptons per baryon of each flavour ($l = e, \mu$), $Y_l = x_l - x_{\bar{l}} + x_{\nu_l} - x_{\bar{\nu}_l}$, are conserved. Gravitational collapse calculations of the iron core of massive stars indicate that, at the onset of trapping, the electron lepton number is $Y_e \simeq 0.4$; since no muons are present at this stage we can impose also $Y_\mu = 0$. For neutrino free matter we just set $\mu_{\nu_l} = 0$ in Eq.(1) and neglect the above constraints on lepton numbers.

We assume a constant value of temperature throughout the core and perform some calculations for a value of temperature ranging from 0 to 50 MeV, with and without neutrinos. The

EOS employed is that of the BHF approach at finite temperature for the hadron phase and that of the MIT bag model for QM. Many more details on the nuclear matter EOS employed together with plots for the chemical composition and pressure at increasing density and temperature can be found in [5].

B. Isentropic envelope

The condition of isothermality adopted for the core cannot be extended to the outer part of the star. Dynamical calculations suggest that the temperature drops rapidly to zero at the surface of the star; this is due to the fast cooling of the outer part of the PNS where the stellar matter is transparent to neutrinos. Moreover, in the early stage, the outer part of a PNS is characterized by a high value of the entropy per baryon ranging from 6 to 10 in units of Boltzmann's constant [1].

For a low enough core temperature ($T \leq 10$ MeV) in [5] a temperature profile in the shape of a step function was assumed, joining the hadronic EoS (BHF) with the BPS [6] plus FMT [7] EoS for the cold crust. When the core temperature T_{core} is greater than 10 MeV, we consider an isentropic envelope in the range of baryon density from 10^{-6} fm^{-3} to 0.01 fm^{-3} based on the EoS of LS [10] with the incompressibility modulus of symmetric nuclear matter $K = 220$ MeV (LS220). Within this EoS, fixing the entropy per baryon s to the above values $(6, 8, 10)$ and imposing beta-equilibrium (neutrino-free regime) we get temperature profiles which rise quikly from 0 (at 10^{-6} fm^{-3}) to values of temperature typical of the hot interior of a PNS (respectively $T_{core} = 30, 40, 50$ MeV) at a baryon density of about 0.01 fm^{-3}. This explicitly suggests a natural correspondance between the entropy of the envelope s_{env} and T_{core} [11].

Since the energy of neutrinos, emerging from the interior, possesses some spreading [8] and their transport properties vary quite a lot during the PNS evolution [9], we do not think that the location of the neutrino-sphere, affected by the same spreading, is a good criterion to fix the matching density between core and envelope. All this instead indicates the possibility to have a blurred region inside the star where we are free to choose the matching density. To build up our model of PNS, a fine tuning of s_{env} is performed in order to have an exact matching between core and envelope of all the thermodynamic quantities (energy density, free energy density, and temperature). In this sense we can consider this static description of PNS with only one free parameter: the temperature of the core. Once T_{core} is chosen, s_{env} is fixed by matching conditions (see [11] for more details and results on pure baryonic PNS).

III. HADRON-QUARK PHASE TRANSITION

We review briefly the description of the bulk properties of uniform QM at finite temperature, deconfined from the beta-stable hadronic matter discussed in the previous section, by using the MIT bag model [12]. In its simplest form, the quarks are considered to be free inside a bag and the thermodynamic properties are derived from the Fermi gas model, where the quark $q = u, d, s$ baryon density and the energy density, are given by

$$\rho_q = \frac{g}{3} \int \frac{d^3k}{(2\pi)^3} \left[f_q^+(k) - f_q^-(k) \right] \tag{2}$$

$$\varepsilon_Q = g \sum_q \int \frac{d^3k}{(2\pi)^3} \left[f_q^+(k) + f_q^-(k) \right] E_q(k) + B \tag{3}$$

where $g = 6$ is the quark degeneracy, $E_q(k) = \sqrt{m_q^2 + k^2}$, B is the bag constant and $f_q^\pm(k)$ are the Fermi distribution functions for the quarks and anti-quarks. We have used massless u and d quarks, and $m_s = 150$ MeV. It has been found [13, 14] that within the MIT bag model (without color superconductivity) with a density-independent bag constant B, the maximum mass of a NS cannot exceed a value of about 1.6 solar masses. Indeed, the maximum mass increases as the value of B decreases, but too small values of B are incompatible with a hadron-quark transition density $\rho > 2$–$3\,\rho_0$ in nearly symmetric nuclear matter, as demanded by heavy-ion collision phenomenology. In order to overcome these restrictions of the model, one can introduce a density-dependent bag parameter $B(\rho)$, and this approach was followed in Ref. [14]. This allows one to lower the value of B at large density, providing a stiffer QM EOS and increasing the value of the maximum mass, while at the same time still fulfilling the condition of no phase transition below $\rho \approx 3\rho_0$ in symmetric matter. In the following we present results based on the MIT model using both a constant value of the bag parameter, $B = 90$ MeV/fm^3, and a gaussian parametrization for the density dependence,

$$B(\rho) = B_\infty + (B_0 - B_\infty) \exp\left[-\beta \left(\frac{\rho}{\rho_0} \right)^2 \right] \tag{4}$$

with $B_\infty = 50$ MeV/fm^3, $B_0 = 400$ MeV/fm^3, and $\beta = 0.17$, see Ref. [14]. The introduction of a density-dependent bag has to be taken into account properly for the computation of various thermodynamical quantities; in particular the quark chemical potentials μ_q and the pressure p are modified as

$$\mu_q \to \mu_q + \frac{dB(\rho)}{d\rho}\,, \quad p \to p + \rho \frac{dB(\rho)}{d\rho}\,. \tag{5}$$

Nevertheless, due to a cancelation of the second term in (5), occurring in relations (1) for the beta-equilibrium, the composition at a given total baryon density remains unaffected by this term (and is in fact independent of B). At this stage of investigation, we disregard possible dependencies of the bag parameter on temperature and individual quark densities. For a more extensive discussion of this topic, the reader is referred to Refs. [14].

The individual quark chemical potentials are fixed by Eq. (1) with $b_q = 1/3$, which implies: $\mu_d = \mu_s = \mu_u + \mu_l - \mu_{\nu_l}$. The charge neutrality condition and the total baryon number conservation together with the constraints on the lepton number Y_l conservation allow us to determine the composition $\rho_q(\rho)$ and then the pressure of the QM phase. In both phases the

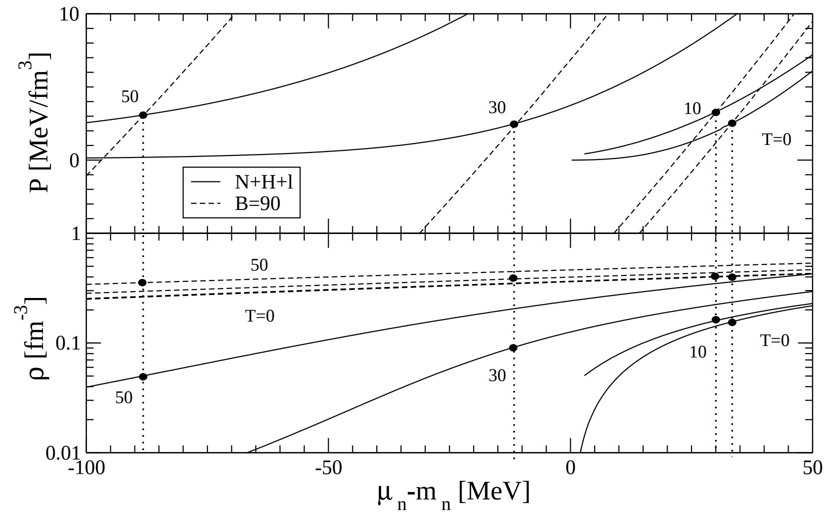

FIG. 1: Baryon density (lower panel) and pressure (upper panel) as a function of baryon chemical potential of beta-stable baryonic matter (solid curves) and quark matter (dashed curves) for the neutrino-free case at different temperatures $T = 0, 10, 30, 50$ MeV. The vertical dotted lines indicate the positions of the phase transitions. A bag constant $B = 90$ MeV/fm^3 is used for QM.

contribution of leptons is that of a Fermi gas. In the range of temperature considered here ($0 \div 50$ MeV) thermal effects are rather weak, the presence of neutrinos instead influences quite strongly the composition: In this case the relative fraction of u quarks increases substantially from 33% to about 42%, compensating the charge of the electrons that are present at an average percentage of 25% throughout the considered range of baryon density, whereas d and s quark fractions are slightly lowered (see [15]).

We now consider the hadron-quark phase transition in beta-stable matter at finite temperature. In the present work we adopt the simple Maxwell construction for the phase transition from the plot of pressure versus chemical potential. The more general Glendenning (Gibbs) construction [16] is still affected by many theoretical uncertainties and in any case influences very little the final mass-radius relations of massive (proto)neutron stars [14]. We therefore display in Figs. 1 the pressure p (upper panels) and baryon density ρ (lower panels) as functions of the baryon chemical potential μ_n for both baryonic and QM phases at temperatures $T = 0, 10, 30, 50$ MeV. The crossing points of the baryon and quark pressure curves (marked with a dot) represent the transitions between baryon and QM phases. The projections of these points (dotted lines) on the baryon and quark density curves in the lower panels indicate the corresponding transition densities from low-density baryonic matter, ρ_H, to high-density QM, ρ_Q.

The main aspects of the EoS for such stellar matter are displayed in Fig. 2 and are well summarized as follows. The transition density ρ_H is rather low, of the order of the nuclear matter saturation density. The phase transition density jump $\rho_Q - \rho_H$ is large, several times ρ_H, and the model with density-dependent bag parameter predicts larger transition densities ρ_H and larger jumps $\rho_Q - \rho_H$ than those with bag constant $B = 90$ MeV/fm^3. The plateaus in the Maxwell construction are thus wider for the former case. Thermal effects and neutrino trapping shift ρ_H to lower values of subnuclear densities and increase the density jump $\rho_Q - \rho_H$. For the cold case the presence of neutrinos even inhibits completely the phase transition [15].

IV. STRUCTURE AND STABILITY

The stable configurations of a PNS can be obtained from the hydrostatic equilibrium equation of Tolman, Oppenheimer, and Volkov [17] for the pressure P and the gravitational mass m, once the EoS $P(\varepsilon)$ is specified, being ε the total energy density. We schematize the entire evolution of a PNS as divided in two main stages. In the first, representing the early stage, the PNS is in a hot ($T_{core} = 30 \div 40$ MeV) stable configuration with a neutrino-trapped core and a high-entropy envelope ($s_{env} \simeq 6 \div 8$). The second stage represents the end of the short-term cooling where the neutrino-free core possesses a low temperature ($T_{core} = 10$ MeV) and the outer part can be considered as a cold crust (BPS+FMT).

The results are plotted in Fig. 3, where we display the gravitational mass M_G (in units of the solar mass $M_\odot$) as a function of the radius R (right panels) and the central baryon density ρ_c (left panels), for QM EOS with $B = 90$ MeV/fm^3 and $B(\rho)$, respectively. Due to the use of the Maxwell construction, the curves are not continuous [16]. PNS in our approach are thus practically hybrid stars and the heaviest ones have only a thin outer layer of baryonic matter. For completeness we display the complete set of results at core temperatures $T = 0, 10, 30, 40, 50$ MeV with and without neutrino trapping, although only the curves with high temperatures and neutrino trapping and low temperatures without trapping are the physically relevant ones. We observe in any case a surprising insensitivity of the results to the presence of neutrinos, in particular for the $B = 90$ MeV/fm^3 case, which can be traced back to the fact that the QM EOS $p(\varepsilon)$ in Fig. 2 is practically insensitive to the neutrino

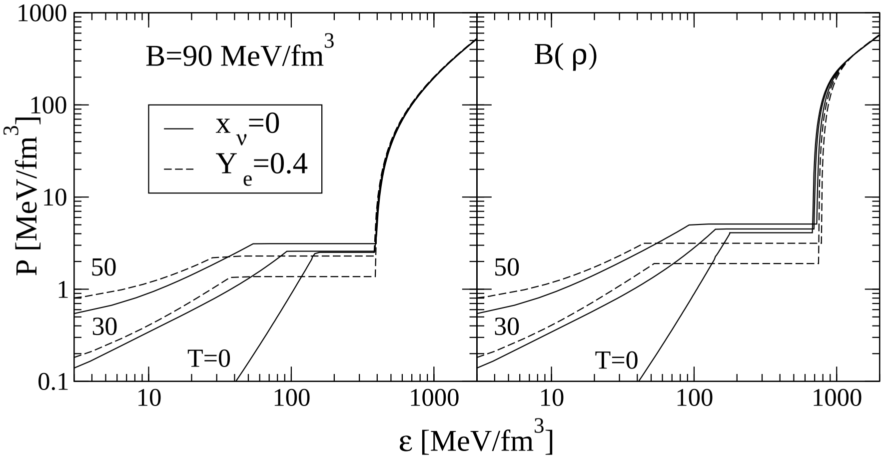

FIG. 2: Pressure as a function of energy density for beta-stable matter with (dashed curves) and without (solid curves) neutrino trapping at different temperatures $T = 0$, 30, and 50 MeV with a bag constant $B = 90$ MeV/fm^3 (left panel) or a density-dependent bag parameter (right panel).

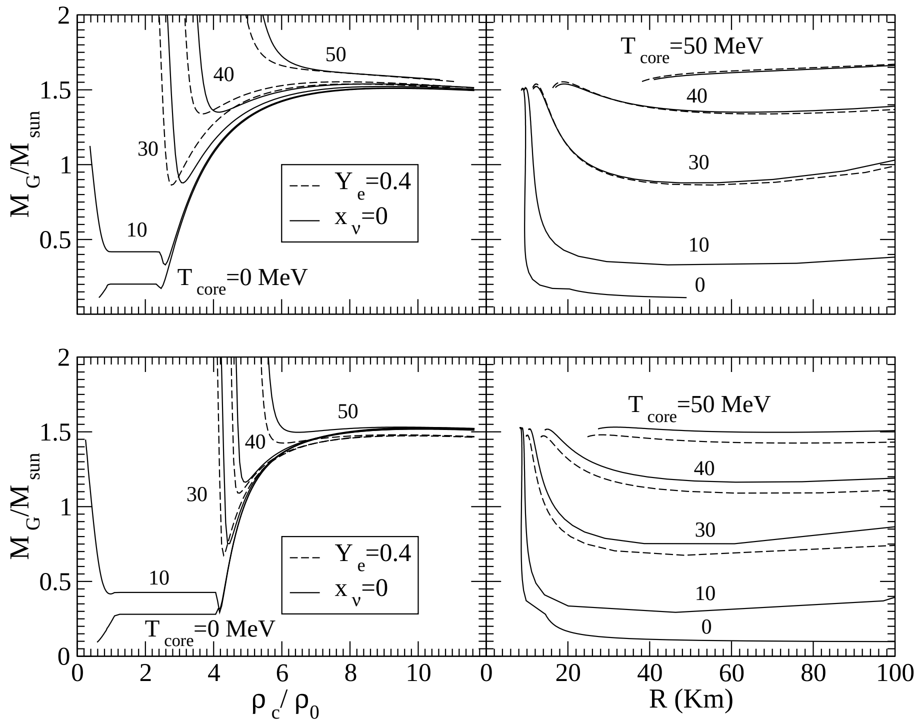

FIG. 3: (Proto)Neutron star mass-central density (left panel) and mass-radius (right panel) relations for different core temperatures $T = 0, 10, 30, 40, 50$ MeV and neutrino-free (solid curves) or neutrino-trapped (dashed curves) matter. A bag constant $B = 90$ MeV/fm^3 is used for QM. Same for lower panels but with a density-dependent bag parameter.

fraction (see [15] for more details). On the other hand, the temperature dependence of the curves is quite pronounced for intermediate and low-mass stars, showing a strong increase of the minimum mass with temperature, whereas the maximum mass remains practically constant under all possible circumstances. Above core temperatures of about 40–50 MeV all stellar configurations become unstable. Concerning the dependence on the QM EOS, we observe again only a slight variation of the maximum PNS masses between 1.55 $M_\odot$ for $B = 90$ MeV/fm^3 and 1.48 $M_\odot$ for $B(\rho)$. Clearer differences exist for the radii, which for the same mass and temperature are larger for the $B = 90$ MeV/fm^3 model.

V. CONCLUSIONS

In conclusion, in this article we have extended a previous work on baryonic PNS [5] to the case of hybrid PNS. We combined the most recent microscopic baryonic EOS in the BHF approach involving nuclear three-body forces and hyperons with two versions of a generalized MIT bag model for QM. The EoS employed for both phases are checked by phenomenological constraints.
We modelled the entropy and temperature profile of a PNS in a simplified way taking as much as possible care about results coming from dynamical calculations [11]. This approach allows us to study the stability of a PNS varying both the temperature of the core and the central density.

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